On the Implementation of Compressive Sensing on Wireless Sensor Network

Dong-Yu Cao, Kai Yu, Shu-Guo Zhuo, Yu-Hen Hu, Fellow, IEEE, and Zhi Wang, Member, IEEE

Abstract—Compressive sensing (CS) is applied to enable real-time data transmission in a wireless sensor network by significantly reduce the local computation and sensor data volume that needs to be transmitted over wireless channels to a remote fusion center. By exploiting the sparse structure of commonly used signals in Wireless Sensor Network (WSN) applications, a Compressed Sensing on WSN (CS-WSN) framework is proposed. This is accomplished by (i) random sub-sampling of data collected at sensor node, (ii) transmitting only the sign-bit of the data samples over wireless channels. It is shown that this CS-WSN framework is capable of delivering similar performance as conventional local data compression method while greatly reduce the data volume and local computation. This proposed scheme is validated using a prototype wireless sensor network test bed. Preliminary experimental results clearly validate the superior performance of this proposed scheme.

Index Terms—wireless sensor array networks, compressed sensing, direction of arrival estimation, data compression.

I. INTRODUCTION

WIRELESS Sensor Networks (WSN) has attracted interests in many applications such as biological studies [1], source localization [2], smart city, disaster forecast, military target tracking [3], etc.

However, dispersed sensors are deployed over large sensor fields and communicate wirelessly to the fusion center. Remote sensor often rely on battery power and are energy constrained. Data samples at remote sensor arrays thus need to be compressed before transmitting to the fusion center to conserve energy consumed for wireless communication. Moreover, executing sophisticated data compression algorithms on remote sensor node will also consume considerable on-board energy. Therefore, the conflict between high-rate, long term WSN application and source constrained sensor node is the key problem remains to be solved.

Generally, WSN platforms are low cost hardware for large scale deployment. The performance of calculation, memory and energy are limited so that only lightweight compression techniques can be applied. On the other hand, the consumption and delay grows exponentially with the data volume since data is transmitted to the center via several hops. Thus the compression ratio should be high to reduce the transmission.

The newly emerged Compressive Sensing (CS) [4] [5] technology exploits the natural sparse property of almost all types of signal and reconstitutes it from greatly reduced random measurement of high dimensional raw signal with high probability. In such case, signal collected with sampling rate much lower than Nyquist rate also works, and the corresponding data volume can be greatly reduced. For the advantage of sampling rate as well as data volume reduction, CS has many application in MRI [6], camera photograph [7], signal acquisition [8], event detection [9], data transmission and other relevant signal processing terms.

Fig. 1. Spectrogram of (a) Porsche engine and (b) bird chirping.

Fig.1 shows the spectrograms of two types of acoustic sources: engine sound of a Porsche vehicle as well as bird chirping [19]. It is easy to observe that these broadband acoustic sources are dominated by multiple harmonics, with additive background noise. The frequency domain sparse structure [20] illustrated in these figures hence may be leveraged to realize compressive sensing.

With the potential benefit of CS technology, someone have tried to introduce CS to WSN framework to eliminate the bottleneck of source restraints on WSN platform. However, the implementation of CS on resource constrained WSN platform is a challenging task. This is because the realization of random measurement also requires digital projection calculation or special hardware design of random projector.

In this paper, we select the typical source localization application using distributed wireless sensor nodes. Instead of transmitting every sensor data to the fusion center, CS technology is utilized to greatly reduce the data volume that need to be transmitted to the remote fusion center wirelessly. Also, the implementation of random measurement of the raw signal in energy constrained sensor nodes are taken into account. We design random sampling and deep quantization scheme [10] [11] that are performed to significantly reduce the amount of data as well as the local processing delay caused by the random measurement scheme. Contrary to traditional data compression algorithm, CS-WSN places computation burden...
at the decoder side and requires little or no computation at the encoder side. Thus, it is very appealing to applications where the sensors are resource constrained. A prototype wireless microphone array platform has been implemented with off-the-shelf devices. Exciting experiment results are presented that convincingly validate the performance advantage of CS-WSN over the state of art alternative algorithms.

The main contributions of this paper can be summarized as follows:

a) A random compressed sampling and a newly emerged 1-bit compressive sensing method are employed.
b) Leveraging the high correlation of acoustic signals among array elements, a collaborative reconstruction scheme is presented to drastically reduce computation cost and delay.
c) Numerical simulation and real experiment on off-the-shelf wireless sensor node validates the feasibility of the implementation on WSN platform.

This paper is organized as follows: Backgrounds of DoA estimation, compressive sensing and 1-bit compressive sensing are reviewed in section 2. CoSCoR framework is presented with details in section 3 and 4. Experiment and simulation results are reported in section 5 along with some analysis. And conclusion is presented in section 6.

II. BACKGROUND

A. DoA Based Array Processing

Assume there are $Q$ sources in the far field that emit plane wave signal $s_q(t)$ consisting of some harmonics. Then the digitized signal received at the $j$th sensor is

$$x_j(t_n) = \sum_{q=1}^{Q} s_{q,0}(nT_s - \tau_{q,j}) + v_j(nT_s), \quad n = 1, 2, ..., N, \tag{1}$$

where $s_{q,0}(t_n)$ is the $q$th acoustic source signal received at a reference sensor node, $\tau_{q,j}$ is the relative difference of propagation delay from the $q$th source to the $j$th sensor and from the same source to a reference sensor node, $v_j$ is the additive Gaussian white noise at the $j$th sensor with zero-mean and variance $\sigma^2$. With above far field assumption, all sensors in the same array should share the same incidence angle $\theta$. It’s easy to verify that the relative time delay of the $q$th source between the $j$th sensor and the array centroid is

$$\tau_{q,j} = \frac{1}{c} (u_j \cos(\theta_q) + y_j \sin(\theta_q)), \tag{2}$$

where $\rho_j = [u_j, y_j]^T$ is the position of the $j$th sensor in the Cartesian coordinate system.

If we represent $x_j = [x_j(t_1), x_j(t_2), ..., x_j(t_N)]^T$ in the discrete Fourier basis $\Psi$, the corresponding Fourier coefficients can be given by

$$x_j = \Psi \alpha, \tag{3}$$

where $\alpha = [x_j(k_1), x_j(k_2), ..., x_j(k_N)]^T$, $x_j(k) = \sum_{q=1}^{Q} S_{q,0} e^{-j \frac{2\pi}{N} \tau_{q,j} \pi k \tau_{q,j}} + V_j(k)$. Consider all the $J$ sensors, the array data spectrum $x(k) = [x_1(k), x_2(k), ..., x_J(k)]^T$ at the $k$th frequency is given by

$$x(k) = \sum_{q=1}^{Q} S_{q,0} \alpha(k) + V(k), \tag{4}$$

where $\alpha(k) = [e^{-j \frac{2\pi}{N} \kappa \pi k \tau_{0,q} \pi / N}, e^{-j \frac{2\pi}{N} \kappa \pi k \tau_{0,q} \pi / N}, ..., e^{-j \frac{2\pi}{N} \kappa \pi k \tau_{0,q} \pi / N}]^T$ is defined as the steering vector corresponding to the $q$th sources at the $k$th frequency.

With traditional high resolution DoA spectral estimation, the sample covariance matrix of array data spectrum $x(k)$ is computed. That is

$$R = x(k)x(k)^H + N\sigma^2I. \tag{5}$$

Exploiting the low rank structure of the signal covariance matrix $R - N\sigma^2I$, numerous DoA methods, such as ESPRIT, MUSIC [12], have been proposed. With the MUSIC method, the DoA of targets are estimated as the peaks of the following function:

$$P_{MUSIC}(\theta) = \frac{1}{a(H(\theta))U_N U_N^{H} a(\theta)}.$$ \tag{6}

where $U_N$ is the noise sub-space after eigenvalue decomposition of $R$, $a(\theta)$ is the steering vector of source.

B. Compressive Sensing and 1-bit Compressive Sensing

Compressive sensing theory [4] [5] states that a sparse signal may be randomly sampled at sub-Nyquist rate and be reconstructed perfectly. Denote $\alpha$ to be a sparse vector representing the sparse signal, $x = \Psi \alpha (x \in \mathbb{R}^N)$ to be the original signal, and $y = \Phi x$ to be the observed signal. According to compressive sensing theory, the observation matrix $\Phi$ can be chosen to be a Gaussian random matrix or Bernoulli random matrix. Given $y$, the CS reconstruction problem is formulated as a constrained optimization problem:

$$\arg \min_{\alpha} ||\alpha||_1, s.t. ||y - \Phi x|| < \eta, \tag{7}$$

where $\eta$ is a preset threshold, and $||\alpha||_1$ is the $l_1$ norm of $\alpha$. The above problem formulation leads to various CS reconstruction algorithms, such as Compressive Sensing Matching Pursuit (CoSaMP), Orthogonal Matching Pursuit (OMP), $l_1$-magic, Lasso and others.

Unlike conventional CS, 1-bit compressive sensing only conserves the sign bit of measurements and discards the magnitude information [10]. In matrix form it is:

$$y = \text{sign}(\Phi x). \tag{8}$$
The 1-bit CS reconstruction optimization problem can be expressed as follows:
\[
\text{arg min}_{\alpha} \|\alpha\|_1 \quad \text{s.t.} \quad Y \Phi \Psi \alpha \geq 0, \quad \|\alpha\|_2 = 1.
\] (9)
The \( \ell_1 \) norm is used as a cost function that enforces the sparseness of \( \alpha \) under the sign constraints. Since the amplitude information is discarded in 1-bit sampling, a unit sphere constraint is introduced to avoid getting the obvious wrong solution of \( \alpha = 0 \).

C. Lossy transmission

In this work, the lossy nature of wireless channel will be exploited by modeling the packet loss during wireless transmission over noisy channel as a form of (involuntary) random data sub-sampling.

Both the random sampling matrix and the random selection matrix will be incorporated as a combined measurement matrix that requires no computation on the sensor nodes while achieving desired data reduction. Denote the random sampling matrix as \( \Phi \), and the random selection matrix over the wireless channel between the sensor node and the fusion center as \( L \). The combined measurement matrix may then be obtained as
\[
\Phi^I = L \cdot \Phi.
\] (10)

III. COLLABORATIVE RECONSTRUCTION

Conventional CS confirms stable signal recovery with the prior information of sparsity. However the sparsity is not the only prior information that helps to recovery signal in sensor array. For spatially neighboring nodes in an array, they receive highly correlated signals, which displays in their similarity in spectrum and difference in phase shift (they share the same support in frequency domain). Thus independent reconstruction of signal at individual sensor seems redundant.

A. Collaborative Reconstruction for Random Measurement

In the fusion center, the received samples of \( J \) sensors can be jointly formulated as:
\[
Y = \Theta A,
\] (11)
where \( Y = [y_1, y_2, \cdots, y_J] \), \( X = [x_1^T, x_2^T, \cdots, x_J^T]^T \), \( \Theta = \text{diag}(\Phi_1 \Psi, \Phi_2 \Psi, \cdots, \Phi_J \Psi) \), \( A = [\alpha_1^T, \alpha_2^T, \cdots, \alpha_J^T]^T \).

Note that each \( x_j \) is a summation of different delay versions of sources signal, \( \alpha_j \) has the same sparse pattern. Based on such joint sparsity, the signal reconstruction problem is
\[
\text{arg min}_{A} \|d\|_1 \quad \text{s.t.} \quad \|Y - \Theta A\|_2 \leq \sigma,
\] (12)
\[\quad d(n) = \sum_{j=1}^J \alpha_j^y(n).\]
Under this joint reconstruction (JoR) model, the theoretical measurement of random CS is \( LK + \log(N/K) \) [13] while the normal CS requires \( cJK \log(N/K) \) [4]. The computational complexity of above joint reconstruction is \( O(J^3N^2) \) [14].

B. Collaborative Reconstruction for 1 bit quantization

Similar to the MMV-prox approach for random measurement, 1 bit quantization also works for joint reconstruction and the joint reconstruction problem can be formulated as:
\[
\text{arg min}_{A} \|d\|_1 \quad \text{s.t.} \quad \text{diag}(Y) \Theta A \geq 0,
\] (13)
\[\quad Y = \text{sign}(\Theta A),
\]
\[\quad d(n) = \sum_{j=1}^J \alpha_j^y(n),
\]
\[\quad |\alpha_j|_1 = 1, j = 1, 2, \cdots, J,
\]
where \( Y = [y_1, y_2, \cdots, y_J] \), \( A = [\alpha_1^T, \alpha_2^T, \cdots, \alpha_J^T]^T \), \( \Theta = \text{diag}(\Phi_1 \Psi, \Phi_2 \Psi, \cdots, \Phi_J \Psi) \).

IV. COMPRESSIVE SENSING ON WSN

The large data volume of array signal is a bottleneck that enables WSAN. Compared with traditional data compression methods, CS performs dimension reduction with projection calculation. In this section, we provide two schemes designed for low-power compressed sampling system. They achieve dimension reduction while requires no extra hardware design or local computation.

A. Random compressed sampling

Realizing random projection in low-power wireless nodes isn’t an easy task. Both the random number generator and projection calculation raises energy consumption and hardware requirement. Inspired by non-uniform sampling method, we combine it with CS framework and model the non-uniform sampling process as a measurement matrix. It is easy to implement and no other computation is needed on nodes.

We design the measurement matrix corresponding to compressive sensing theory as below: The \( M \times N \) measurement matrix \( \Phi \) represents the measurement process, each row has only one nonzero component 1 in the \((m, t_m)\) position. The physical nature of the ADC is such that we take samples at \( t_m \). Let a sequence \( u = \{u(1), u(1), \cdots, u(M)\} \) be such that \( u(1) = 1 \), and
\[
u(m) = u(m - 1) + [\tau_m], \quad 2 \leq m \leq M,
\] (14)
where \( \tau_m \in N \left( \frac{M^2K^2}{N} \right) \) is the random sampling interval between adjacent sample instance. Then the \((m, n)\)th element of this proposed random sampling matrix is given by
\[
\Phi(m, n) = \begin{cases} 1 & 1 \leq n = u(m) \leq N; \\ 0 & \text{otherwise.} \end{cases}
\] (15)
As such, no random projection calculation is required in our sensor nodes. This is important to the sensor nodes, because both random matrix generation and random projection requires large computational and energy resource. Sensor works at \( f_s \), then randomly selects subset of samples and transmits them via wireless links.

B. 1-bit Compressive sensing

On condition that the sampling rate and energy consumption can be support in sensor node, 1-bit CS is an appealing
solution to the data volume reduction problem mentioned above. And there are two means of realization:

a) Using hardware of random number generators and comparator. After random projection of raw signal, a comparator to zero is utilized to quantize the measurement. In fact, comparator to zero is an extremely cheap and fast device.

b) Perform random projection in software. ADC samples at the Nyquist rate, and then the sign of projected raw signal is conserve. Such method has been used in 1-bit Sigma-Delta converters, at the expense of high sampling rate [15].

V. SYSTEM OVERVIEW

In this section, a detailed description of the hardware system is provided. We take REXENSE STM32W108 hardware testbed as the sensor node prototype system, as shown in Fig. 3. An IEEE 802.15.4 based wireless module is integrated into the chip, and a serial port for data stream input. Some parameters are described in Table I.

![Wireless sensor prototype system](image1)

**Fig. 3.** Wireless sensor prototype system

<table>
<thead>
<tr>
<th>Features</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clock Speed</td>
<td>24MHz</td>
</tr>
<tr>
<td>RAM</td>
<td>12kB</td>
</tr>
<tr>
<td>ROM</td>
<td>128kB</td>
</tr>
<tr>
<td>Wireless Protocol</td>
<td>IEEE 802.15.4</td>
</tr>
<tr>
<td>Power of Wireless Module</td>
<td>100mW</td>
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</table>

For general 16bit 2kHz audio signal, the data rate is 4kB/s. However, the theoretical maximum data rate of IEEE 802.15.4 is 250kbps, which is too slow for audio signal transmission. When deploy the audio sensor nodes in large scale, the network will be very congested and the delay will increase sharply. To solve the problem, we propose a compressive sensing based data compression application layer for WSAN to reduce the data transmission volume.

The first step is to generate a sampling matrix and store it into the memory of the MCU. However, the widely used Gaussian random sampling matrix occupies too much space that cannot be stored in the RAM or ROM. In addition, it is impossible to generate the matrix dynamically either because the calculation consumption is too large. Consider the limitation of local computational and local storage, conventional data compression algorithm such as MP3, LZW, Huffman can not be implemented in low power SOC chips.

Compared to Gaussian matrix, the Bernoulli matrix saves more space since each element of it is a boolean value rather than a float in Gaussian matrix.

A better choice of sampling matrix is the random subsampling matrix. In this matrix, there is only one non-zero element in each row. Thus we can record the position of the non-zero element and the space consumption is equal to the number of row. Moreover, the projection process takes little time since there is no add and multiply operation in random subsampling.

Based on random subsampling, we proposed a novel one-bit random subsampling method for further reducing the transmission volume.

VI. PERFORMANCE EVALUATION

In this section, extensive simulations are carried out to compare the proposed CS-WSN framework against traditional array processing approach. A hardware prototype is developed to validate the practical application of the CS-WSN approach through outdoor experiments.

A. Experiment and Simulation Settings

Theoretically, our schemes work for any array formation when it satisfies the array spacing requirements. A uniform linear array with 6 nodes and a −90° to 90° target space is chosen for both simulation and experiment. Considering that the acoustic signal of cars, trucks or helicopters is usually dominated by a few harmonics, the source signal is assumed to be summation of harmonic at 500Hz, 600Hz, 700Hz and 800Hz. Accordingly, a 0.2m spacing is adopted to satisfy the half-wavelength requirements of array. We choose the MUSIC algorithm for DoA estimation. The dimension of original signal (also reconstructed signal) in a snapshot is assumed to be 512, and the system sampling rate is set to be 2048Hz. In this section, we denote RSS as random subsampling, RBS as random Bernoulli measurement, 1BS as 1 bit Sampling, and RSSJR, RBSJR, 1BSJR are corresponding joint sparse based reconstructions. To have fair comparison, we formulate the random compressive sensing and 1 bit CS based approach as an SOCP problem and solve them using Sedumi toolbox [16].

B. Parameters setting

The first issue of CS in real application is the number of random measurement. Although some theoretic bounds has been proposed, it is still not clear that how many samples will bring maximum benefit. Therefore the SNR of reconstructed signal is chosen as criteria. For 1 bit CS that only reconstruct the waveform, the coherence between raw signal and reconstructed signal is used as criterion that indicates the similarity between raw signal and reconstructed signal. Fig. 4 and Fig. 5 show the relationship between the reconstructed SNR and the corresponding measurements $M$. Table II shows the measurement number that reaches original 10dB.

Table II shows the optimal measurement number for all kinds of CS-based approaches proposed in this paper. The optimal measurement number is the threshold that confirm a stable signal reconstruction. It is easy to observe the joint
sparsity model using a MMV-prox formulation enables stable reconstruction performance with greatly reduced measurement number. Here $r_{dc}$ is defined as the data reduction ratio between compressed sampled data and raw data.

C. Experiments description

In this experiment, we use 7 testbeds to evaluate the Multi-Point-to-Point (MP2P) performance of the CS based sampling techniques. We mainly focus on the multi-hop delay without packet loss and reconstruction accuracy with packet loss.

![Fig. 6. Experiment Scenario and Network Topology](image)

Six testbeds work as sensor node to form an acoustic sensor array. One testbed works as router for packet relay, and another testbed works as sink for packet collection. Two acoustic sources are placed 20 meters away from the sensor nodes. The angles of acoustic sources to center of sensor array are $-5.5^\circ$ and $31^\circ$. The data length of a data frame is 64B in hop-1. The router collects the packets from nodes, regroups the data and sends to the sink. The packet length is different in hop-2. For directly sending method, the length is 64B. For Bernoulli sampling and sub-sampling, it is 48B. For 1-bit Sampling, it is 6B. The network works in TDMA mode.

Fig. 7 shows the MP2P delay comparison of different sampling methods. In multi-hop network, the delay increase extremely (560ms) without compression due to the large data volume, but a little with CS based methods. The main reason is that the data volume is deeply reduced with CS based sampling method. Using Bernoulli Sampling technique, the delay is reduced to 170ms. It takes less time using sub-sampling and 1-bit sampling.

The impact of measurement number of three CS-based approach are studied in experiment. We change the measurement number from 10 to 120 (interval = 10) of RB-mmV and RS-mmV and from 40 to 360 (interval = 40) of 1BSJR to test the performance. From Fig. 8 we can see that with the increase of measurement number, compressive sensing methods have a better performance of signal reconstruction accuracy. The RSSJR works better than RBSJR at most measurement numbers, and 160 bit confirms a stable reconstruction for 1BSJR.

![Fig. 9. DoA estimation error with different measurement number (a) Source 1 (b) Source 2](image)

The DoA estimations using reconstructed signals are compared in 9 and 10. It is easy to observe that RSSJR and

<table>
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<th>TABLE II</th>
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<tr>
<td><strong>NUMBER OF MEASUREMENT</strong></td>
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<td></td>
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<tr>
<td>RBS</td>
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<td>RSS</td>
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<td>1BS</td>
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RBSJR work has the same DOA performance under the same data volume. The 1BSJR achieve acceptable performance with deepest data reduction. This is because the magnitude information lost in the 1 bit quantization procedure does not violate the DOA estimation, the reconstructed waveform provides enough information for array processing.

We conduct another experiment to evaluate the influence over lossy link. Six sensor nodes are placed near the router, while the sink is placed far from the router so that the link between the router and sink can be considered as lossy link in which packet loss happens randomly. We change the distance between router and sink to obtain different packet loss rate.

Fig. 11 illustrates the reconstruction accuracy and SNR of different CS based sampling method over lossy link. When using CS based sampling techniques, the signal can be reconstructed precisely even in low packet received rate scenarios. The accuracy of Bernoulli sampling and sub-sampling decline in a higher ratio than 1-bit sampling, since 1bcs-mmv has the deepest data reduction ratio.

Fig. 12 shows the DOA Estimation Error of CS based sampling methods over lossy link. For source 1, the DoA estimation error remains low with the PRR more than 0.7. As PRR decreases, the DoA estimation error varies sharply and randomly. At the same time, the DoA estimation error of source 2 remains low even in low PRR. The Bernoulli sampling gives the best performance of DoA estimation accuracy.

VII. CONCLUSION

In this paper, a CS-WSN framework is proposed. By exploiting the sparse nature of source signal, random compressed sampling and 1-bit sampling could sample much less data while retaining acceptable performance. By exploiting the high correlation among array signal, collaborative reconstruction effectively reconstruct array signal as well as further reduce the number of samples required for non-reference nodes.

Both experiments on hardware and numerical simulation are presented to validate the usefulness of the proposed CS-WSN framework. It is shown that CS-WSN works well especially under harsh data limitations. Considering the convenience of implementation on hardware, CS-WSN would be an excellent choice on low-cost, low-power wireless array network.

ACKNOWLEDGMENT

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REFERENCES


Fig. 10. DoA estimation error of 1 bit CS with different measurement number (a) Source 1 (b) Source 2

Fig. 11. Signal Reconstruction Accuracy over Lossy Link

Fig. 12. DoA estimation error over lossy link (a) Source 1 (b) Source 2