Pseudo-Haptic Display of Mass and Mass Distribution
During Object Rotation in Virtual Reality
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Abstract—We propose and evaluate novel pseudo-haptic techniques to display mass and mass distribution for proxy-based object manipulation in virtual reality. These techniques are specifically designed to generate haptic effects during the object’s rotation. They rely on manipulating the mapping between visual cues of motion and kinesthetic cues of force to generate a sense of heaviness, which alters the perception of the object’s mass-related properties without changing the physical proxy. First we present a technique to display an object’s mass by scaling its rotational motion relative to its mass. A psycho-physical experiment demonstrates that this technique effectively generates correct perceptions of relative mass between two virtual objects. We then present two pseudo-haptic techniques designed to display an object’s mass distribution. One of them relies on manipulating the pivot point of rotation, while the other adjusts rotational motion based on the real-time dynamics of the moving object. An empirical study shows that both techniques can influence perception of mass distribution, with the second technique being significantly more effective.

Index Terms—Virtual reality, object manipulation, object rotation, human perception, pseudo-haptics

1 Introduction
Haptic sensation is an important part of a virtual reality (VR) experience [5, 20]. It helps us understand the (simulated) physical aspect of a virtual environment (VE) and is critical for enhancing the sense of presence [12, 14, 34, 47]. However, providing effective haptic feedback during interactive object manipulation in VR is difficult. In object manipulation, modern commercial VR systems often let the user hold a proxy (called a hand-held controller), and pick up different virtual objects using this same tool [44]. A particular proxy provides the same haptic stimuli for all virtual objects, which contradicts the scenario of holding different objects with distinct mass properties. Thus, an item that is intended to be heavy may suddenly feel as light as the proxy once it is picked up [45]. Deviations from the user’s expectation (like this mismatch of heaviness) could reduce the VE’s coherence [36-38, 40], potentially leading to decreased plausibility [39, 40]. This problem can be addressed through active haptic rendering, which directly displays haptic forces in real-time (e.g., through a robotic arm attached to the user’s hand), but such approaches are often cumbersome and expensive [13, 45].

Apart from haptic stimulation, visual stimuli can also influence the perception of heaviness (e.g., [33]). This phenomenon has been leveraged in VR through pseudo-haptic techniques that change the user’s perception of virtual objects without altering the physical proxy [18]. For example, varying the control/display (C/D) ratio of an object’s translation directly influences the perception of its total mass [7]. A larger C/D ratio makes objects feel heavier, since the same hand movement results in less visual movement of the object. This technique has been applied in VR settings to successfully alter mass perception with the user holding the same physical proxy [31, 35]. Similar approaches of scaling visual movement during rotational motion have also proven effective, but only in highly controlled settings [41, 42]. It remains to be seen whether this approach can be applied to rotation in a VR setting with unconstrained input. This paper intends to fill this gap by taking a close look at how rotational motion could be used to generate mass-related illusions in VR interaction.

In this paper, we first investigate whether scaling an object’s rotational movement will influence the perception of its total mass relative to other objects. We present statistical evidence from a psycho-physical experiment to support this hypothesis. Beyond the perception of an item’s total mass, we recognize that object rotation is also affected strongly by the distribution of mass within an object. For example, swinging a golf club where most of the mass is concentrated in the club head feels very different than swinging a metal rod made of a single homogeneous material, even if both have the same total mass. With this observation, we discuss the design and implementation of two pseudo-haptic techniques to display mass distribution within an object. One of them relies on manipulating the pivot point of rotation, while the other adjusts the C/D ratio on the fly based on the real-time dynamics of the object’s movement. Our experiment shows that both techniques can influence perceptions of mass distribution, with the second technique being much more effective.

As far as we are aware, this is the first effort focusing on pseudo-haptic display of mass and mass distribution during unconstrained rotational control in VR. The techniques presented here are designed to be easily applicable to common proxy-based manipulation in VR systems. The contributions of this paper include:

- Empirical evidence that changing the C/D ratio of rotational motion in VR effectively influences the user’s perception of an object’s total mass.
- Two novel pseudo-haptic techniques for displaying mass distribution during rotation in VR, along with empirical evidence that at least one of these techniques is very effective in creating such effects.

2 Related Work
Before introducing pseudo-haptic techniques for generating mass-related perceptions in immersive VEs, we discuss research describing how humans fundamentally perceive heaviness. Existing research has suggested that the perception of heaviness is a multimodal one, influenced by both haptic and visual information. Charpentier’s size-
mass illusion was a demonstration of such effect [24]. This illusion occurs when the weight of a larger item is underestimated compared to a smaller one with the same mass. Amazeen et al. later suggested that the true source of this illusion is not the change in size, but instead the change in the object’s rotational inertia, which describes how mass is distributed over the volume [1]. Masin et al. conducted an experiment that asked participants to look at an object before, during or after lifting it, and the influence from vision was only found when they could see it while lifting [23]. This result suggests that the object’s motion might be a more crucial factor compared to its static appearance.

Other studies (also outside the context of VR) have shown that heaviness can be effectively perceived by visual information alone. Runeson et al. demonstrated that the weight of an item can be accurately judged by watching a point light display of a person lifting it [33]. Similar effectiveness of visual information on mass perception was also shown in an experiment by Bingham [4]. Runeson et al. proposed the “kinematic specification of dynamics (KSD) principles”, which explains how dynamics (force and mass) could be interpreted through kinematics (motion) using the physics-based causality between the two [32, 33]; the novel algorithms presented in this paper are based on a similar physics-based model.

Closely related to the experiments in this paper, Streit et al. tested how haptically perceived rotational inertia interacts with visual displacement of motion in generating heaviness perception [41, 42]. In these experiments, participants were asked to rotate a handle attached to a fixed frame while watching the manipulated motion on a 2D screen. The result suggests that perception of heaviness can be consistently affected by both sensory channels, and this influence conforms to the physics-based correlation between kinematics and dynamics.

The aforementioned studies were not performed in immersive VE’s, but the multimodal nature of haptic perception has been recognized and leveraged for desirable effects in VR. Since building a matching physical prop for every virtual object in a VE is not practical in most situations, one line of research explores the practicality of mapping multiple virtual items to one physical prop without the user noticing it, which is achieved by exploiting the dominance of visual information to alter haptic sensations. Kohli et al. proposed the concept of “redirected touching” [15] and found that this manipulation could have minimal effect on task performance [16] and users could adapt to it very well in a training task [17]. Azmandian et al. proposed “haptic retargeting” [2] and Lohse proposed “haptic change blindness” [21] to accomplish similar effects. Zemen et al. reported the detection thresholds when the user’s hand is displaced visually, which could be applied for haptic retargeting purposes [46]. Fujinawa et al. described a novel method for computationally designing a hand-held controller so its haptic properties could convincingly represent a virtual counterpart even though they look different [8].

The interaction between visual and haptic perception has been leveraged to influence many types of haptic sensations in VR, creating a research area called “pseudo-haptics” [28]. It has found successful applications in changing the perception of friction [19], mass [7, 31, 35], shape [3], stiffness [10], softness [27], torque [25] and force [29]. Pseudo-haptic display of mass is usually achieved by visually displacing the hand and/or the object, so a sense of heaviness is generated though kinesthetic cues [7, 31, 35]. Research also finds that such a technique could be used to control fatigue during object lifting [43].

Regarding the perception of an object’s mass distribution, Lukos et al. [22] found that people change how they grab an object (in the real world) if they can visually predict its center of mass (CoM). We assume that this will also hold true in VR, which is strong motivation to develop techniques that adjust rotational motion based on an object’s mass distribution. Hashiguchi et al. used diminished reality to hide part of a real rod to influence the perception of its mass and mass distribution [11]. Zenner et al. developed a “weight-shifting dynamic haptic proxy” that could change its mass distribution on the fly [45], which directly addresses the challenge of displaying mass distribution by introducing novel hardware that combines active and passive haptic display. Experiments found that this device leads to a more realistic and enjoyable experience compared to an equivalent proxy with fixed mass distribution. Zenner’s thesis also provides a thorough overview of materials related to this issue [44]. Similar to this, Cheng et al. proposed a liquid-based haptic proxy that can dynamically change its mass properties, but no formal evaluation was conducted [6]. Our work attempts to avoid the need for special-purpose hardware by displaying mass distribution using pseudo-haptics alone.

3 Approach

In object manipulation, the object’s motion is causally linked to the force exerted onto it [32, 33]. For rotation, this correlation can be expressed in the rotational version of Newton’s second law:

$$\tau = I \alpha$$ (1)

In this equation, \(\tau\) is the torque, which is the rotational analog of force. The overall torque of object manipulation can come from both hand input and the object’s gravitational force. \(I\) is the moment of inertia, which describes mass-related properties of the object; and \(\alpha\) is the angular acceleration, which concerns the object’s motion. \(I\) is the dynamics of the system, which is perceived haptically through muscular tension, while \(\alpha\) is the kinematics, which is seen visually by its movement. According to the KSD principle [32, 33], scaling up the visual motion (\(\alpha\)) while maintaining the same haptic cues (\(\tau\)) would imply a decrease in \(I\) and vice versa. We leverage this interplay between the two sensory channels to manipulate the perception of the virtual object’s mass.

Compared to translational motion, what is unique about rotation is that both the haptic information (\(\tau\)) and mass-related characteristics (\(I\)) in (1) are only meaningful when tied to a specific rotational axis, and different axes will result in different values for them. \(\tau\) is defined as a vector generated by the cross product of the radius vector (from the axis of rotation to the point of application of force) and the force vector, and \(I\) is the sum of the products of each particle’s mass in the rigid body with the square of its distance from the rotational axis (for a more thorough explanation of the dynamics of rigid body rotation, please refer to [9]). In addition to the object’s total mass, how mass is distributed relative to the axes matters to both of these concepts by definition. Thus, when discussing rotation, we cannot treat the entire object as a point mass, but rather we need to understand its mass distribution relative to the rotational axis. Consider again the golf club example: the forces required to rotate the golf club change drastically depending on where it is held and which direction it is rotated. This special feature of rotational motion gives us an opportunity to not only manipulate the perception of an object’s total mass, but also the perception of how mass is distributed at a finer level of granularity.

It is important to point out that we treat total mass and mass distribution as two separate physical properties in designing the pseudo-haptic techniques in this paper. On the one hand, two objects can have the same shape and total mass but distinct characteristics in mass distribution; on the other hand, two objects can have the same shape and the same relative mass distribution between different parts of the body, but different total masses (e.g., because one object is made of a different material). In sections 4 and 5, we describe techniques for generating these two illusions and evaluate their effectiveness. In the paper’s conclusion, we will discuss how these two aspects of pseudo-haptics can be combined for practical use.

4 Pseudo-Haptic Display of Total Mass During Rotation

4.1 The Technique

All the interaction techniques in this paper are based on the common “simple virtual hand” metaphor, in which a zero-order mapping maps the hand-held controller’s movement to the object’s movement (translation and rotation) [26]. We use the “remote control” version of
this metaphor, which maps the hand’s relative movement to the movement of a remotely placed object. This is similar to how we use a mouse to drag icons in a desktop environment, which does not require the hand and the object to be co-located. The reason we use remote manipulation is to enhance the applicability of the proposed techniques—not requiring the physical proxy and the virtual object to be co-located makes them compatible to other pseudo-haptic techniques that are based on displacing the virtual object from the proxy (e.g., [7, 31, 35]), which inherently results in remote control.

Our technique for producing a pseudo-haptic display of total mass during rotation is in direct analogy with the technique of changing C/D ratio in translation [7]. It’s also very similar to how rotational motion is scaled to influence perceived heaviness in the experiments by Streit et al. [41, 42]. We change the C/D ratio for rotation based on the object’s mass—a heavier object will have a larger C/D ratio (i.e., its rotational motion is scaled down) and vice versa. The C/D ratio is applied to rotational motion using several steps. First, for every frame, the orientation (originally expressed as a quaternion) of the hand-held proxy is extracted. Then the difference in the orientation between the last frame and the current frame is used to calculate an axis and an angle, which represent the proxy’s relative rotation between these two frames. The appropriate C/D ratio is applied to scale the angle while keeping the same axis. Finally, these values are used to rotate the virtual object.

This technique loosely conforms to the KSD principle under equation (1). Assuming the pivot point is at the CoM, the only active torque in the system is the torque from hand input, since torque due to gravity becomes zero if the rotational axis goes through the CoM. As long as the mass distribution is constant, increasing the C/D ratio has the effect that the same exerted torque from the hand will result in less visual rotation of the object, which in turn increases the total mass of the specified I in (1). The reason why we say this is a loose conformation is that changing C/D ratios relate to changes in velocity, not acceleration. But this approach appears to be “close enough” to effectively influence the perception of relative mass between objects.

4.2 Experiment I

We designed a psycho-physical experiment to evaluate the effectiveness of this technique. The experiment tests whether a user can perceive a mass difference between two virtual objects rotated with different C/D ratios.

4.2.1 Experimental Design

Figure 1 shows the virtual environment used in this experiment. In this scene, two cubes with the same appearance (size, shape, and texture) were placed side by side. The user was asked to rotate them in turn and make a two-alternative forced choice about which of them felt heavier than the other. The rotational control of the two cubes was scaled by difference C/D ratios. If the technique is effective, we expected that users would perceive that objects rotating with larger C/D ratios would feel heavier than the other, and vice versa. In this experiment, the pivot point of rotation was always at the geometric center of the cube.

Figure 1: The virtual environment for experiment I.

For each trial, the system randomly picked one of the cubes to assign it a non-1:1 rotational C/D ratio, while the other cube had a ratio of 1:1 (used as the reference). The levels of the first C/D ratio are listed in Table 1 (note that the table lists the reciprocal of C/D ratios), with values centered around 1.0. We can think of these values as being linearly scaled by the virtual mass of the items. For example, assuming the reference cube with a 1:1 C/D ratio has a mass of 1kg, another cube with a mass of 5kg would have a C/D ratio of 1:0.2 (calculated by 5kg/1kg).

Table 1: Scaling values used in experiment I (values represent the reciprocal of C/D ratio)

<table>
<thead>
<tr>
<th>rotation scaled down</th>
<th>rotation scaled up</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.1</td>
</tr>
<tr>
<td>0.5</td>
<td>1.3</td>
</tr>
<tr>
<td>0.7</td>
<td>1.5</td>
</tr>
<tr>
<td>0.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

One thing worth noting is that this experiment only evaluates if different rotational C/D ratios could lead to the perception of one item being heavier than the other. It does not evaluate the perception of the absolute value of the object’s mass or the actual mass ratio between two objects. Using the previous example, even though we set the C/D ratio based on the objects’ virtual mass, we don’t claim that using a C/D ratio of 1:0.2 could lead to the perception that the item weighs 5kg or that it feels 5 times heavier than the 1kg reference cube. It only tests if the one with a C/D ratio of 1:0.2 feels heavier than the reference with 1:1. We consider the perception of the absolute values of mass and mass difference as future work.

To eliminate any influence from translational motion or the translational C/D ratio (which has already been shown to effectively produce a pseudo-haptic illusion of mass perception [7]), both virtual cubes were fixed in place and only had three degrees-of-freedom (DOF) of rotation. In the real world, the user held a physical proxy in her dominant hand, with its rotational motion mapped to the currently selected cube using the assigned C/D ratio. Another hand-held controller in the non-dominant hand was used to select one of the cubes using ray-casting, as shown in Figure 3A. The user indicated which cube felt heavier by clicking on the button next to the corresponding cube (Figure 1) using the same ray.

4.2.2 Procedure and Measures

With three repetitions for each of the eight C/D ratio levels shown in Table 1, each participant made 24 total mass judgments. The order of the trials was randomized. For each trial, if the cube with a larger C/D ratio was selected as the heavier one, we considered it as a “correct” response. For each participant, we measured the percentage of correct responses for each level of C/D ratio as the primary quantitative measure.

Using correct responses alone potentially puts us at risk that we were only measuring participants’ ability to guess the correct answer instead of actually measuring their perception—users may have guessed that the object that rotates slower “should” be the heavier one, but they may not have been experiencing an actual sensation of mass difference. To alleviate this issue, we also adopted a post-experiment questionnaire on the quality of induced haptic perception, proposed by Rietzler et al. [30]. Customized to fit the context of this experiment, the questionnaire asked participants how much they agreed with the following six statements on a seven-point Likert scale (expressed to the user using the labels “strongly disagree,” “disagree,” “somewhat disagree,” “neither agree nor disagree,” “somewhat agree,” “agree,” or “strongly agree”):

Q1: I could feel that the two cubes have different weight
Q2: The representation of weight difference felt realistic
Q3: I had the feeling of manipulating real objects
Q4: The representation of weight difference is sufficient for VR
Q5: I liked this representation of weight difference
Q6: I felt I have control

In the questionnaires used in this paper, we exchanged the term “mass” with “weight”. Even though they have different meanings in physics and technically “mass” was the more accurate term here, we felt “weight” was a more understandable expression for a general audience and could bring up what we were looking for in the questionnaires all the same, namely the subjective sensation of heaviness when manipulating these virtual objects.

Each participant first went through an introduction session, which explained the task and the interface. Next, we provided a training
session to ensure that the participants could understand the experimental procedure, interact with the virtual scene, and complete the two-alternative forced choice tasks. This training session included two trials with one of the cubes exhibiting downscaled rotation using the C/D ratio of 1/0.2, and another two trials with one cube showing upscaled rotation using the C/D ratio of 1/1.8. After the training, each participant completed the 24 experimental trials. During both the training and the experiment, no feedback was given after each trial; instead, the system simply played a sound indicating the conclusion of the current trial and immediately loaded the next one. Finally, we gave the participant the post-experiment questionnaire.

4.2.3 Apparatus and Participants

Participants sat in a chair to complete the experiment. A wireless HTC VIVE Pro head-mounted display was used to display the virtual environment (Figure 2A). This headset has a resolution of 1080 x 1200 pixels per eye (2160 x 1200 pixels combined), a refresh rate of 90 Hz and a field of view of 110 degrees. The user used a VIVE Pro controller (Figure 2C) in her non-dominant hand to select between the two cubes and give answers using ray-casting, while a VIVE tracker (Figure 2B) was used in her dominant hand to rotate the virtual cubes. The VE displayed a virtual representation of the VIVE Pro controller (and the ray emitted from its tip) so the user could easily select items with ray-casting. The VIVE tracker itself was not displayed since visually seeing its orientation could potentially make it apparent that the cube’s rotation was scaled. The headset, controller, and tracker were all tracked by an HTC Lighthouse 2.0 inertial-optical tracking system. The interface and environment were developed in Unity 2018.3.2f1. Rendering was performed by a PC with an Intel Core i7-8700k CPU, 16GB of RAM, and an Nvidia GTX 1070 GPU.

Figure 2: A. the user used the VIVE tracker to manipulate the object while holding the VIVE Pro controller to give answers and select between the two cubes in the first experiment. B: The VIVE tracker. C: The VIVE Pro controller.

A key aspect of the experiment was the choice of the physical proxy, as its physical shape and mass could potentially bias mass-related perception. Since experiment I was about distinguishing the overall mass of two virtual objects that look identical, as long as the same physical proxy is used in the same way when manipulating the two cubes, the shape and mass of the proxy should not bias the user’s choice. On the other hand, it is desirable to choose a proxy that is small and light enough to be easy for the user to hold and rotate. We ended up using the VIVE tracker (Figure 2B) as it is lightweight and easy to manipulate with one or two hands.

Throughout the experiment, the participant sat at a table, so he could put the controller down if he wanted to use both hands to manipulate the VIVE tracker (Figure 2A). The table could also provide elbow support if needed to mitigate potential fatigue.

A total of 17 participants (4 females, 13 males) were recruited, all between 20 and 30 years old. Among them, only two of them identified their left hand as the dominant hand. Experiment II (Section 5.3) also used this same group of participants.

4.2.4 Results

Figure 3 shows the correctness results. As the figure shows, participants achieved almost 100% correctness at all levels of C/D ratio except those close to 1.0. Even at these levels, the average correctness was greater than 80%.

We treat the correctness percentage as ordinal data. For C/D ratios of 1/0.2, 1/0.7, 1/1.3, 1/1.5, 1/1.8, the answers were 100% correct so no statistical test was needed. For the rest of the cases, one-sample Wilcoxon signed rank tests show that the results are all significantly higher than random chance of 50%, with p-values of 4.103e-14 (C/D = 1/0.5), 0.0001 (C/D = 1/0.9) and 0.0003 (C/D = 1/1.1).

Figure 3: Percentage of correct responses at each C/D ratio in experiment I. Error bars represent standard deviation.

Figure 4 shows the results from the questionnaire. One-sample Wilcoxon signed rank tests show that the mean responses are significantly higher than 0 for all questions, with p-values of 0.0004 (Q1), 0.0006 (Q2), 0.0008 (Q3), 0.0002 (Q4), 0.002 (Q5), 0.0001 (Q6).

Figure 4: Box plot of subjective questionnaire results in experiment I. Mean values are indicated by an ’x.’

4.3 Discussion

The results of experiment I strongly support the effectiveness of the C/D ratio technique for producing a pseudo-haptic effect making one virtual object seem heavier than another. Not only could participants robustly identify the heavier object, even with small mass differences (Figure 3), but also they affirmed the technique’s power through the questionnaire, in which they indicated that the mass difference felt strong (Q1), realistic (Q2, Q3), sufficient (Q4) and desirable (Q5) (see
Figure 4). As mentioned in Section 4.2.2, although the identification of the heavier cube only demonstrates that the user could correctly guess the result, the positive response of the questionnaire here indicates that the illusion was indeed perceived as subjective sensations of mass differences. The results from Q6 of the questionnaire also show that adjusting the rotational C/D ratio did not compromise the perceived controllability. Overall, this technique and experiment complement the previous works by Dominjon et al. that used translational C/D ratio to generate the perception of mass [7] and the experiments by Streit et al. [41, 42] that used constrained rotation. Based on these results, adjusting the C/D ratio for both translation and rotation in VR object manipulation should generate a convincing illusion of mass variation.

Figure 3 shows that users’ ability to perceive mass differences decreases as the C/D ratio gets closer to 1.0. A limitation of this current experiment is that, due to our choice of C/D ratio levels, we were not able to find the threshold of mass difference that will lead to detectability close to random chance. We consider finding that threshold as part of the future work.

5 PSEUDO-HAPTIC DISPLAY OF MASS DISTRIBUTION DURING ROTATION

5.1 Techniques

Recognizing that mass distribution also plays an important role in rotational motion, we leverage the same interplay between kinesthetic torque cues and visual movement cues to display pseudo-haptic mass illusions at a finer level of granularity. We propose two different techniques, both leveraging the intuitive relationship between torque, rotational motion and mass expressed by (1).

5.1.1 Pivot Technique (PV)

The first technique, called “the pivot technique,” simply sets the pivot point of rotation at the object’s CoM while maintaining a constant C/D ratio for rotation around any axis. We expect that users will be able to naturally feel where the CoM is when it coincides with the rotational pivot point. Figure 5 illustrates this idea in 2D: imagine that the virtual object is said to be made of the same material but does not have even density across its body. Starting from the upper-left corner, the density gradually becomes greater as we move towards the lower-right corner. With this mass distribution, the CoM will lie close to the lower-right corner instead of at the geometric center. Using PV, the object rotates around this CoM, as illustrated by Figure 5.

Figure 5: Illustration of the PV technique. The object rotates around the CoM instead of the geometric center.

In remote object manipulation, we expect the user to feel that he is gripping the object right at the pivot point of rotation. Hence, PV creates the illusion that the user is always gripping the object at its CoM. Intuitively, this is how people tend to grab an object in the real world, because gravity does not create any torque relative to the CoM, making the object more controllable [22]. PV is designed so that users see a constant rotational speed (C/D ratio) around the pivot point, indicating that this point is the CoM, and thus giving the user information about the object’s mass distribution.

This technique also loosely conforms to the KSD principles following equation (1). According to (1), the overall torque of the system should be influenced by gravity, which in turn should affect the rotational speed (C/D ratio), unless the pivot coincides with the CoM (where torque due to gravity does not exist). Thus, keeping a constant C/D ratio specifies that there is no torque due to gravity, which means that the CoM is at the pivot. Note that the C/D ratio should be technically be adjusted according to the direction of rotation since I could change according to this direction. However, for simple objects like cubes (used in the experiment described in Section 5.2) that rotate around the CoM, the slight change of I would have little influence, so we decided to ignore this factor to simplify the technique and keep a constant C/D ratio.

5.1.2 Dynamic Torque Technique (DT)

One limitation of PV is that it requires modifying the pivot of rotation. But in practical VR applications, it is preferred that users can grasp objects at any point, motivating the need for a more general-purpose technique. The second technique, called “dynamic torque,” achieves by adjusting the rotational C/D ratio using the real-time mechanics of the rotating object given an arbitrary pivot.

The general idea is shown in Figure 6. Assume we are using the same square-shaped virtual object in which the CoM is at the bottom right, but the rotational pivot is now at the geometric center rather than at the CoM. This means that gravity will generate a torque relative to this pivot (shown in Figure 6 as a vector pointing into the 2D plane). This torque affects how difficult it is to rotate the object based on its relationship with the vector of the rotational axis. If they lie in the same direction (the angle between them is smaller than 90 degrees, as in the case of the clockwise rotation in Figure 6), the torque due to gravity will aid the intended rotation. Hence, we scale up the rotational motion using a smaller C/D ratio in this case. Conversely, if the torque due to gravity is opposed to the direction of the rotational axis, we scale down the rotation with a larger C/D ratio, as in the counterclockwise rotation in Figure 6.

Figure 6: Illustration of the DT technique. The torque due to gravity is a vector pointing into the 2D plane (marked ‘-’). It affects the C/D ratio based on the axis and direction of rotation.

DT requires that rotational motion should be scaled according to the direction of rotation at each instant. Thus, we need to adjust the rotational C/D ratio on a frame-to-frame basis. In the following paragraphs, we derive how this ratio can be computed for each frame in a way that is both feasible and intuitive.

For each frame, the so-called rotational C/D ratio is the ratio between the amount of rotation of the physical proxy (Aph) and the amount of rotation of the virtual object (Av). Given a desired C/D ratio and Aph measured in the current frame, we can compute Av.

First we recognize that there are two sources of torque in this system: the torque from hand input (τhand) and the torque from gravity (τgg). We re-write (1) into (2):

\[ τ_{hand} + τ_{gg} = Iα \]  

We turn this equation into a general relationship between torque, mass distribution and rotation that loosely follows the KSD principle:

\[ τ_{hand} + τ_{pp} \approx I_{pp}A_{p} \]  

In (3), τhand is the torque from hand input, representing how much kinesthetic effort the user is exerting to rotate the object in that frame. The value of this component should be determined by how the hand-held proxy is being rotated. The other torque component, τpp, is the specified virtual torque generated by gravity acting on the object’s mass, projected onto the rotational axis. Note that since we are using
a zero-order mapping between the physical proxy’s movement and the virtual object’s movement, the axis of rotation for the object is solely determined by the rotation direction of the proxy in that frame. The torque from gravity should not influence this direction; it should only influence the amount of rotation. This reduces the problem from 3D rotation to 2D rotation. Thus, torque due to gravity is projected onto the rotational axis.

On the right side of (3), $I_v$ is the specified moment of inertia of the virtual object relative to the rotational axis. Again, $A_v$ is the amount of rotation for the virtual object, which is the value we are looking for.

Next, we reform the general relationship of (3) into an equation by giving each component an empirically determined constant ($e_0, e_1, e_2$):

$$e_0 \text{hand} + e_1 \tau_{gp} = e_2 I_v A_v$$  \hspace{1cm} (4)

We approximate the kinesthetic effort from hand input ($\tau_{\text{hand}}$) by the product between the physical proxy’s moment of inertia ($I_{\text{ph}}$) and how much it was rotated in the current frame ($A_{\text{ph}}$):

$$e_0 I_{\text{ph}} A_{\text{ph}} + e_1 \tau_{gp} = e_2 I_v A_v$$  \hspace{1cm} (5)

To solve for $A_v$, we re-organize (5) into (6):

$$A_v = (e_0 I_{\text{ph}} A_{\text{ph}} + e_1 \tau_{gp})/e_2 I_v$$  \hspace{1cm} (6)

In this formula, all the components on the right side can be computed ($I_{\text{ph}}, \tau_{gp}, I_v$), empirically set ($e_0, e_1, e_2$), or measured in real-time from the user’s input ($A_{\text{ph}}$).

Directly using (6) may cause a problem: the torque due to gravity can counter the hand’s intended rotation while having a very large value, which could result in a rotation angle of zero or in the opposite direction of the hand’s input. This would break the zero-order mapping of the control. To overcome this issue, we set a minimum scale for the object’s rotation to be 10% of the physical proxy’s rotation (a maximum C/D ratio of 1/0.1), so the zero-order mapping from hand input to the object’s movement is always maintained. Thus, we use the following formula to calculate the angle the virtual object should rotate in each frame:

$$A_v = \max \left( (e_0 I_{\text{ph}} A_{\text{ph}} + e_1 \tau_{gp})/e_2 I_v, 0.1 \cdot A_{\text{ph}} \right)$$  \hspace{1cm} (7)

In the next section, we show the values used in the computation of (7) based on the specific virtual object used in the experiment.

Figure 7: Left: VE for experiment II. Right: the four possible CoM positions in experiment II.

5.2.4 Experimental Design

To evaluate the effectiveness of the two proposed techniques, we designed an experiment to test whether the mass distribution displayed through these techniques could be correctly perceived.

5.2.4.1 Experimental Design

The experimental task was to manipulate the yellow cube shown in Figure 7, left, which is composed of eight sub-cubes at the corners. Among these sub-cubes, one is heavier than the other seven (with these other seven having the same mass). The heavier sub-cube is randomly picked by the system and unknown to the user, since the sub-cubes all have the same appearance. The system uses either $PV$ or $DT$ to display the mass distribution, and the user must judge which sub-cube is heavier after manipulating the object (an eight-alternative forced choice task).

By adjusting the ratio between the mass of the heavier sub-cube and the lighter ones, the CoM of the entire box will move along the diagonal of the heavy sub-cube (shown in two dimensions in Figure 7, right). The larger this ratio is, the closer the CoM is placed to the center of the heavy sub-cube. We set the total mass of the cube at 100g and used four levels of mass ratio to create four conditions, as shown in Table 2. These values were chosen so that the positions of the CoM would be evenly spread on the diagonal, as shown on the right side of Figure 7. For each condition, $PV$ used the corresponding CoM position as the rotational pivot, while $DT$ used the corresponding values in Table 2 to drive its computation of $A_v$ using formula (7).

To carry out the actual calculation of (7), several values on its right side need to be computed or empirically set. First, we use a constant value to approximate $I_{\text{ph}}$ based on the size and mass of the VIVE tracker: $I_{\text{ph}} = 0.0004 \text{ kg m}^2$.

Table 2: Mass ratio conditions in experiment II.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Heavy sub-cube mass</th>
<th>Light sub-cube mass</th>
<th>Heavy/light ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30g</td>
<td>10g</td>
<td>3:1</td>
</tr>
<tr>
<td>2</td>
<td>47.5g</td>
<td>7.5g</td>
<td>6.33:1</td>
</tr>
<tr>
<td>3</td>
<td>65g</td>
<td>5g</td>
<td>13:1</td>
</tr>
<tr>
<td>4</td>
<td>82.5g</td>
<td>2.5g</td>
<td>33:1</td>
</tr>
</tbody>
</table>

The virtual cube has a size of 0.1m x 0.1m x 0.1m, with each sub-cube having a size of 0.05m x 0.05m x 0.05m. Given the mass properties (Table 2) and size of the object, $\tau_{gp}$ and $I_v$ can be calculated using their definitions in physics given a rotational axis [9]. Here, we briefly introduce how these values are computed.

$\tau_{gp}$ is computed by first calculating the torque due to gravity $\tau_g$ using the cross product between the radius vector $r$ (the vector from pivot to CoM) and the object’s total gravitational force (marked as $M_v g$ for the big cube):

$$\tau_g = r \times M_v g$$  \hspace{1cm} (8)

Then this vector of $\tau_g$ is projected onto the unit vector of the rotational axis to compute $\tau_{gp}$.

The computation of $I_v$ is a little more complicated. For a composite object such as the big cube we used, $I_v$ can be generated by adding the moment of inertia of each individual component. Hence, $I_v$ is the sum of the moment of inertia from each sub-cube (marked as $I_{vi}$):

$$I_v = \sum_{i=1}^{n} I_{vi}$$  \hspace{1cm} (9)

To compute each $I_{vi}$, the parallel axis theorem is applied. This theorem states that, if we know an object’s moment of inertia when it is rotating about an axis passing through its CoM (marked as $I_{vi,\text{CoM}}$ for each sub-cube), we can further deduce its moment of inertia when it’s rotating around any axis that’s parallel to the original one. Such calculation uses the distance between the two axes ($d$) and the mass of the object (marked as $m_v$ for each sub-cube):

$$I_{vi} = I_{vi,\text{CoM}} + m_v d^2$$  \hspace{1cm} (10)

For our cube object, $m_v$ is listed in Table 2, $d$ can be computed using the distance from each sub-cube’s geometric center (also the CoM for each sub-cube) to the current rotation axis. $I_{vi,\text{CoM}}$ is generated using the following formula no matter the direction of rotation, with $s$ being the length of the sub-cube’s edge:

$$I_{vi,\text{CoM}} = \frac{1}{6} m_v s^2$$  \hspace{1cm} (11)

Finally, we need to set the constants $e_0, e_1, and e_2$. These values determine the influence of each component of (7). Enlarging $e_0$ will...
increase the influence of the physical proxy’s rotation, so the same kinesthetic effort from the hand will result in more rotation of the virtual object. Enlarging $e_1$ will increase the influence of torque due to gravity, so the resulting C/D ratio will fluctuate more dramatically when the rotational axis changes. Enlarging $e_2$ will increase the influence of the moment of inertia of the virtual object, so that it is more reluctant to move. After empirical testing, we set the following values: $e_1 = 1; e_2 = 0.02; e_3 = 1$.

Note that since the display of mass distribution is independent from the display of total mass (explained at the end of Section 3), the overall scale of the C/D ratio for both translation and rotation should not affect the perception of mass distribution. One can uniformly scale the C/D ratio of translation (Dominjon et al. [7]) and/or rotation (Section 4) to display the object’s total mass while still using either PV or DT to display the mass distribution. Thus, we do not need to separate translation from rotation in this experiment (unlike experiment I, in which we allowed only rotation, since the C/D ratio of translation would affect the perception of total object mass). To maximize the potential applicability of these techniques, we gave the user full six DOF control in this experiment.

5.2.2 Procedure and Measures

This experiment used a within-subject design—each participant used both techniques in turn for the same set of tasks. The order of presentation was counter-balanced. For each technique, there were four repetitions for each of the four conditions (Table 2), resulting in a total of 32 trials (4 repetitions $\times$ 4 conditions $\times$ 2 techniques) per participant. For each technique, the order of the 16 trials was randomized. For each trial, the position of the heavy sub-cube was also randomized.

The participant indicated which sub-cube was perceived to be heavier by selecting the corresponding numbered button (Figure 7, left) using ray-casting. Each participant’s percentage of correct answers was used as the primary quantitative measure. Note that since this is an eight-alternative forced choice, random chance would lead to a 12.5% correctness rate.

We also included the questionnaire [30] used in experiment I to evaluate the quality of the haptic sensation. Modified to match the context of this experiment, the questionnaire asks the participant to rate agreement with these statements on the same 7-point Likert scale:

Q1: I could feel that one cube is heavier than the other seven
Q2: The representation of weight distribution felt realistic
Q3: I had the feeling of manipulating a real object
Q4: The representation of weight distribution is sufficient for VR
Q5: I liked this representation of weight distribution
Q6: I felt I have control

We also asked participants to indicate which of the two techniques they preferred according to the same criteria. Specifically, we asked the following questions:

Q1: Which technique gave you a stronger sense of weight distribution (one of the sub-cubes is heavier than the others)?
Q2: Which representation of weight distribution felt more realistic?
Q3: Which technique gave you a sensation that’s closer to the feeling of manipulating real objects?
Q4: Which representation of weight distribution felt more sufficient for VR applications?
Q5: Which technique do you prefer?
Q6: Which technique felt easier to control?

Participants first went through a tutorial session, in which the concept of mass distribution and the experimental task were explained. Similar to experiment I, participants first went through a training session to practice the task four times with the first technique. All of these four trials used the 33:1 ratio as the mass ratio between the heavy sub-cube and each of the light sub-cubes (condition 4 in Table 2). After the training session, the participant was asked to complete the 16 experimental trials. The system did not give any feedback on correctness during the practice or experimental trials. After using the first technique, participants completed the

![Figure 8: Percentage of correct responses by each technique in each mass ratio condition in experiment II. Error bars represent standard deviation.](image-url)
confirmed that DT significantly outperformed PV in the first three conditions when the mass ratio was 3:1, 19:3 or 13:1, but not the last condition when this ratio was 33:1 (p = 0.008, 0.011, 0.044, 0.068 respectively for conditions 1-4). We also compared the result of each technique in each condition against random chance (12.5%) using one-sample Wilcoxon signed rank tests with Bonferroni correction. Correctness levels for DT were significantly higher than random chance in all conditions (p = 0.002, 0.001, 0.001, 0.001 for conditions 1-4). However, PV outperformed random chance significantly only when the mass ratio was 13:1 and 33:1 (p = 2.044, 0.264, 0.010, 0.001 for conditions 1-4). These results are annotated in Figure 8.

Figure 9 shows the results from the individual technique questionnaire. Wilcoxon signed rank tests showed that DT had significantly higher scores than PV on Q1, Q2, Q4 and Q5. The p-values are 0.008 for Q1, 0.027 for Q2, 0.386 for Q3, 0.043 for Q4, 0.042 for Q5 and 0.503 for Q6. Responses for DT were significantly higher than 0 for all the questions, with p-values of 0.0002, 0.008, 0.001, 0.003, 0.008 and 0.005 for Q1-Q6. In the case of PV, the results are only significantly higher than 0 on Q3 and Q6. The p-values are 0.128, 0.862, 0.0002, 0.606, 0.833 and 0.0005 for Q1-Q6. These results from statistical analysis are annotated in the figure.

Figure 10 shows the results from the preference questionnaire. Except that slightly more participants believed that PV is easier to control than DT, more people preferred DT on all other criteria.

5.3 Discussion

The results of experiment II show that DT is a powerful technique in generating the perception of mass distribution. When using DT, participants could usually perceive which part of the object was heavier even when the CoM was only slightly off the geometric center. Evaluation of subjective user experience also shows that the effect was perceived to be strong (Q1), realistic (Q2, Q3), and sufficient (Q4), and that the technique was appreciated (Q5). The fact that a non-ideal proxy was used (Section 5.2.3) further demonstrates the power and applicability of this technique. Overall, this empirical evidence shows the high effectiveness of DT in producing a pseudo-haptic display of mass distribution.

PV, on the other hand, seems to be much weaker in generating this illusion. Even though participants gave correct answers at a significantly higher rate than random in two of the four conditions, these conditions had very high mass ratios (13:1 and 33:1). Participants using PV did not find the illusion to be strong (Q1), realistic (Q2, Q3), sufficient (Q4), or desirable (Q5), and it was generally considered as the sub-optimal choice when compared to DT. This could be seen in both the individual evaluation of PV (Figure 9) and the comparison between the two techniques (Figure 10).

Both techniques were considered to be very easy to control (Q6), and slightly more participants thought PV was more controllable. This is not a surprise, as PV maintains the 1:1 rotational C/D ratio throughout, while DT changes it on the fly. When controllability is at high priority, PV may still have some value even though its pseudo-haptic illusion seems to be much weaker.

A limitation of the DT technique is that there are several empirically determined constants in our algorithm. These were specifically tweaked to fit the current experiment, and they may need to be further modified when using DT for a new VE. Another limitation is that the influence from the mass distribution of the physical proxy is not explored in this experiment, and we consider it as part of the future work. A limitation of the current experimental procedure is that we cannot be sure that the fixed order of the two experiments had no impact on the result; future studies could validate our findings without this ordering limitation.

6 Conclusions and Future Work

This research contributes several pseudo-haptic techniques to influence mass-related perceptions during rotational motion in VR object manipulation. Experiment I showed that one can display a difference in total masses by scaling objects’ rotational C/D ratios. In addition, we also found empirical evidence that by altering the C/D ratio dynamically based on the physical principles that explain the motion of rotation, we can effectively display certain aspects of mass distribution. These two pseudo-haptic techniques can be applied simultaneously—scaling the base C/D ratio to alter the perception of total mass while adjusting this base C/D ratio dynamically relative to itself to display mass distribution.

The techniques can also be seamlessly combined with translation-based pseudo-haptic techniques to further enhance the display of mass-related properties. In any VR application requiring realistic object manipulation, these techniques can be directly applied, which
could potentially enhance plausibility and the overall sense of presence. Such effects could also have a positive impact on the enjoyment and engagement of the VR experience, which are crucial for certain applications such as entertainment, training and psychotherapy.

In the future, we plan to study the physical proxy’s shape, mass and mass distribution, to see how these factors could affect the strength of the pseudo-haptic illusion. A follow-up study should also determine the threshold when the mass difference between the two objects become indistinguishable. Moreover, we’d like to investigate how the parameters used in these techniques connect to the perceived absolute values that quantify mass, mass difference and mass distribution. To continue this line of research, we also plan to combine these two pseudo-haptic effects with translation-based techniques and study their use in more realistic virtual environments, which could potentially identify measurable benefits on broad user experience from adding these pseudo-haptic illusions (e.g., on perceived presence, fun and engagement).

**REFERENCES**


